NIUS Lecture Notes

On

Quantum Computation (SPIN)

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Lecture notes on

Quantum Computation

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1 The Electron Spin

1.1 Classical Considerations

Gyromagnetic ratio Consider a charge q orbiting in a circle of radius R and with time period $2\pi/\omega$. The magnetic moment of the charge in motion is

$$\mu = iA = \frac{q\omega}{2\pi}\pi R^2 = \frac{q}{2}\omega R^2$$

The angular momentum of the motion is

$$L = mR^2\omega$$

$$(Momentofinertia \times angularvelocity)$$

$$\therefore \mu = \frac{q}{2m}L$$

This relationship can also be obtained for a spinning disc or sphere on whose surface the charge q is uniformly smeared.

For the electron, we have for its spin, $L = \hbar/2$

$$\mu_B = g\frac{q}{2m}\frac{\hbar}{2}$$

$$\text{where} \quad g = 2.00231930436153$$

$\therefore$ the magnetic moment of the electron is

$$|\mu_B| = \frac{e\hbar}{2m} = \frac{1.6\times10^{-19}\times6.626\times10^{-34}}{2\times9.1\times10^{-31}\times2\pi}$$

$$= 9.27\times10^{-24}J/T$$

$$(= 5.79\times10^{-5}eV/T)$$

Electron in external magnetic field The magnetic moment experiences no force in a unifrom magnetic field. But t does experience a torque. Newton's II law in this case is

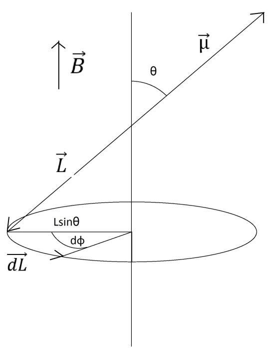

Figure 1: Angular momentum posses around making constant angle with it

$$\frac{d\vec{L}}{dt} = \vec{\mu}\times\vec{B} = \frac{q}{2m}\vec{L}\times\vec{B}$$

$$\therefore L^2 \text{ is a constant quantity.}$$

It is not hard to see that both $\vec{L}$ and $\vec{B}$ are perpendicular to $\frac{d\vec{L}}{dt}$. Further, the magnitue of $\vec{L}$ is a constant, for

$$\vec{L}\cdot\frac{d\vec{L}}{dt} = \vec{L}\cdot\vec{L}\times\vec{B}\left(\frac{q}{2m}\right) = 0$$

$$\therefore \frac{1}{2}\frac{d(\vec{L}\cdot\vec{L})}{dt} = 0$$

Another point to note is that $\vec{L}$ makes a constant angle with $\vec{B}$.

$$|d\vec{L}| = L\sin\theta d\phi$$

$$|d\vec{L}| = Ldt = \frac{q}{2m}LB\sin\theta dt$$

$\therefore$ equating,

$$\frac{d\phi}{dt} = \frac{q}{2m}B = \omega_L$$

ω_L is the larmor precession frequency.

For the electron in a field of say 1 T,

$$\omega_L = \frac{q}{2m}B \Rightarrow \nu_L = 5\times10^10Hz$$

$$Energy, \hbar\omega_L = 3.13 \times 10^{-23} J = 2.07 \times 10^{-4} eV$$

Energy of the magnetic moment carrying electron in external magnetic field,

$$U = -\vec{mu} \cdot \vec{B} = \frac{|e|\hbar}{2m} B$$

Force is only possible if the magnetic field is non-uniform. It is

$$\vec{F} = -\vec{\nabla} U = \vec{\nabla}(\vec{mu} \cdot \vec{B})$$

For simple case, $\vec{F} = \mu_z \frac{dB}{dz}$

1.2 The Stern-Gerlach Experiment

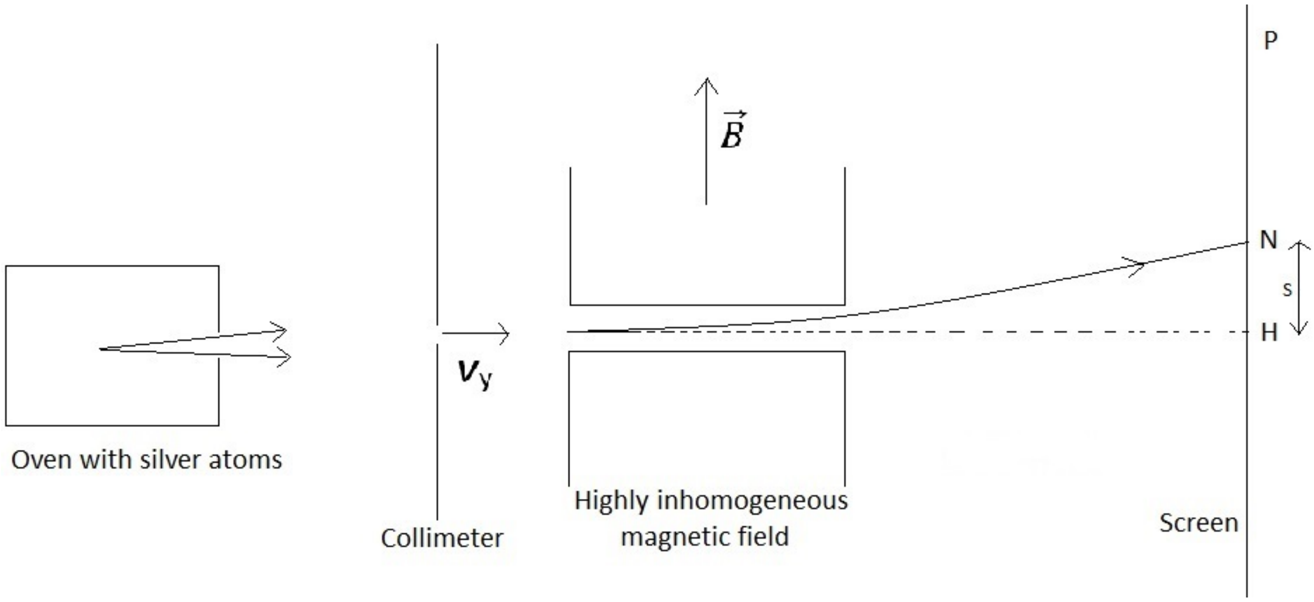

Figure 2: Schematic of Stern-Gerlach Experiment

$$s = \frac{1}{2} a t^2 \quad \text{(no gravity)}$$

$$v_y t = l \quad \text{(say l extends all the way upto the screen)}$$

$$s = \frac{1}{2} a \frac{l^2}{v_y^2}$$

$$a = \frac{F}{m_{Ag}} = \frac{\mu_z \frac{dB}{dz}}{m_{Ag}}$$

$$s = \frac{1}{2} \frac{l^2}{v_y^2} . \mu_z \frac{dB}{dz} . \frac{1}{m_{Ag}}$$

Why Silver? Angular momentum = spin of the outer electron $d^1 0 s^1$.

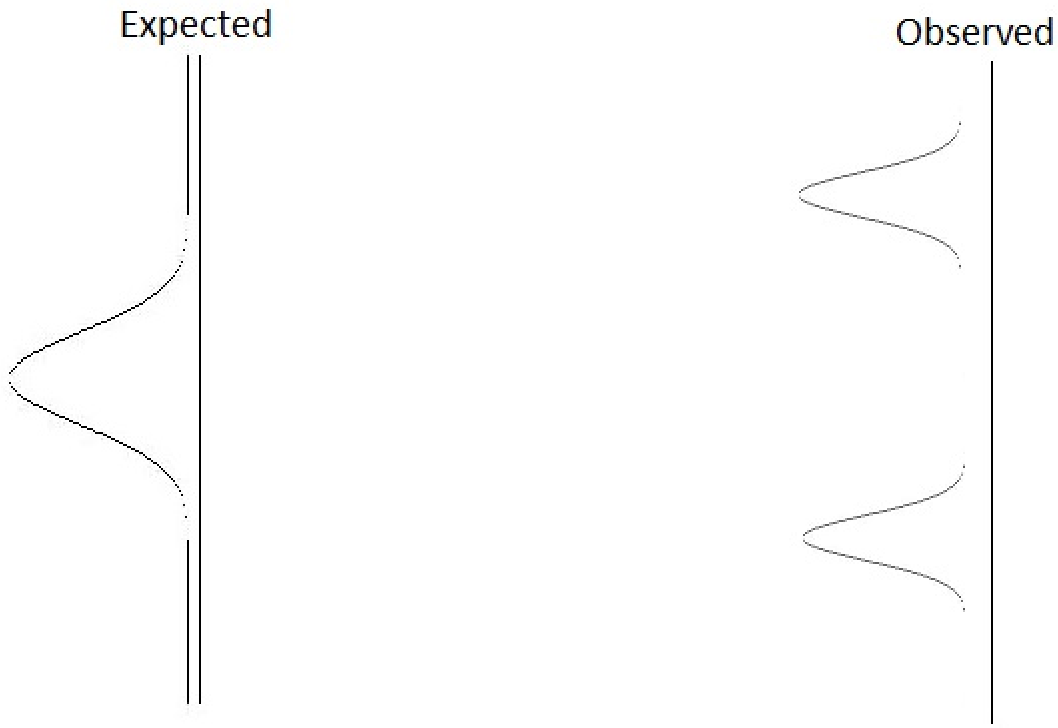

Figure 3: Expected and Observed distribution of atoms on the screen

Since on emerging from the oven, the magnetic moments of the silver atoms are isotropically distributed between $\mu_B\hat{k}$ $-\mu_B\hat{k}$, we expect that the atoms will fall on the screen forming a gaussian. However, the observed result is that all the atoms are deflected to two particular positions on the screen.